

# TE<sub>10</sub> Mode Scattering by a Rectangular Resistive Film of Arbitrary Dimensions Placed Along the Rectangular Waveguide Axis

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**Abstract** — TE<sub>10</sub> mode scattering by a resistive film of arbitrary width ( $d$ ) and length ( $l$ ) placed in the longitudinal section of a rectangular waveguide parallel to its narrow faces is investigated. The vector integral equation for the discontinuity ( $\vec{h}$ ) of the tangential magnetic field on the film is formulated. The equation is solved by Galerkin method using basis functions, each of them taking into account the  $\vec{h}$  behavior near the film edge. For a film that is sufficiently short ( $l \ll d$ ), approximate expressions for the scattering matrix elements are obtained. The scattering matrix for a wide range of values of the film width, length, surface impedance ( $W$ ), and frequency is calculated. This is believed to be the first study establishing that the attenuation caused by a film having particular  $d$  and  $W$  values tends to be constant over the entire band of waveguide operating frequencies.

## I. INTRODUCTION

THE performance of most attenuating microwave devices [1] is based on the principle of electromagnetic wave power absorption by a resistive film (RF) placed in the plane of the electric field vector and wave propagating direction. In a waveguide with a rectangular cross section, the RF is placed along the waveguide axis and parallel to its narrow faces. The problem of TE<sub>10</sub> mode diffraction by the RF in the longitudinal plane of a rectangular waveguide was solved in [2]–[6]. References [2]–[4] deal with the semi-infinite RF while [5] and [6] concern themselves with an RF of finite length. All these studies assumed that the width  $d$  of the film coincides with the waveguide height  $b$  and that the RF is in electric contact with its surface. Such a diffraction problem is scalar and two-dimensional and it may be reduced to a one-dimensional integral equation [4], [6].

When the RF width is less than waveguide height, the problem becomes very complicated; i.e. it is vectorial and three-dimensional. In spite of an RF with width  $d < b$  being applied in variable attenuators [1], this diffraction problem has never been investigated. Our paper presents calculations of the scattering characteristics for the RF shown in Fig. 1 as well as investigations of the physical properties of the given discontinuity.

## II. STATEMENT OF THE PROBLEM

It is assumed that in the region  $z = -\infty$  of the rectangular waveguide (Fig. 1) there is an incident TE<sub>10</sub> mode with electric field

$$\vec{E}_0(x, z) = U \sin(\alpha_1 x) \exp(ik_{10}z) \vec{a}_y \quad (1)$$

where  $k_{mn} = \sqrt{k^2 - \alpha_m^2 - \beta_n^2}$ ,  $\alpha_m = m\pi/a$ , and  $\beta_n = n\pi/b$ . In addition  $k$  is the wavenumber in free space,  $U$  is the dimensional factor, and  $\vec{a}_x$ ,  $\vec{a}_y$ , and  $\vec{a}_z$  are the unit vectors directed along the  $x$ ,  $y$ , and  $z$  axes. The time dependence of the fields is presented as  $\exp(-i\omega t)$ . Since the incident mode field is not dependent upon  $y$  and the configuration in Fig. 1 is symmetrical with respect to the  $y = b/2$  plane, only TE<sub>mn</sub> and TM<sub>mn</sub> modes with even subscript  $n$  are excited. We assume that  $b \ll a$  and  $\pi < ka < 2\pi$ . From these considerations, it arises that, among all the modes being excited in the  $|z| > l/2$  regions, there is only one propagating mode.

The scattering problem is reduced to the vector integral equation [7]

$$W\vec{h}(\vec{p}) + \int d\sigma' \vec{\zeta}(\vec{p}, \vec{p}') \cdot \vec{h}(\vec{p}') = [\vec{a}_x \times \vec{E}_0(\vec{p})] \quad (2)$$

for the discontinuity  $\vec{h} = \vec{H}|_{x=x_0-0} - \vec{H}|_{x=x_0+0}$  of the magnetic field  $\vec{H}$  on the RF and to the calculation of the scattering matrix elements

$$\begin{aligned} S_{11} = S_{22} &= -\frac{\zeta_0 k}{Uk_{10}ab} \exp(ik_{10}l) \sin(\alpha_1 x_0) \\ &\cdot \int d\sigma h_z(\vec{p}) \exp(ik_{10}z) \\ S_{21} = S_{12} &= \exp(ik_{10}l) \left[ 1 - \frac{\zeta_0 k}{Uk_{10}ab} \sin(\alpha_1 x_0) \right. \\ &\left. \cdot \int d\sigma h_z(\vec{p}) \exp(-ik_{10}z) \right]. \end{aligned} \quad (3)$$

Here  $W$  is the surface impedance of the RF,  $\vec{\zeta}$  is the wave impedance tensor [7],  $\vec{p}$  is the radius vector of the point on the RF,  $\zeta_0 = 120\pi \Omega$  is the wave impedance in free space,

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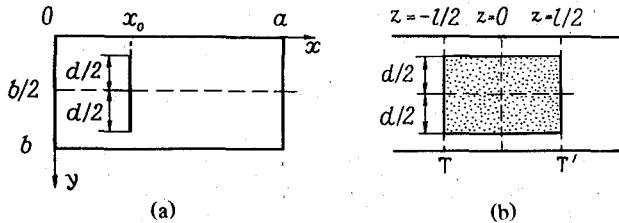


Fig. 1. Rectangular waveguide with the RF in longitudinal plane.  
(a) cross section; (b) longitudinal section.

\$d\sigma\$ is an element of the RF surface. The reference planes \$T\$ and \$T'\$ for the scattering matrix are shown in Fig. 1(b).

The components of the tensor \$\zeta\$ may be presented in the integral series form

$$\zeta_{rs}(\vec{p}, \vec{p}') = -\frac{i\zeta_0}{2\pi kb} \int dp \sum_{n=0}^{\infty} \epsilon_n F_{2n}(p) R_{2n}^{rs}(p; y, y') \exp[ip(z - z')] \quad (4)$$

$$F_n(p) = \frac{\sinh[\sqrt{p^2 - k_{0n}^2}(a - x_0)] \sinh[\sqrt{p^2 - k_{0n}^2}x_0]}{\sqrt{p^2 - k_{0n}^2} \sinh[\sqrt{p^2 - k_{0n}^2}a]} \quad (5)$$

$$R_n^{rs}(p; y, y') = \begin{cases} (k^2 - p^2) \sin(\beta_n y) \sin(\beta_n y'), & r = s = y \\ ip\beta_n \sin(\beta_n y) \cos(\beta_n y'), & r = y, s = z \\ -ip\beta_n \cos(\beta_n y) \sin(\beta_n y'), & r = z, s = y \\ (k^2 - \beta_n^2) \cos(\beta_n y) \cos(\beta_n y'), & r = s = z \end{cases} \quad (6)$$

where \$\epsilon\_n = 2 - \delta\_{0n}\$, \$\delta\_{mn}\$ being the Kronecker delta. Integration with respect to the complex variable \$p\$ is performed across the real axis from \$-\infty\$ to \$\infty\$. During this integration the poles placed on the negative (positive) semiaxis are encircled clockwise (anticlockwise) along a small semicircumference. When \$b \ll a\$ and \$\pi < ka < 2\pi\$, \$F\_{2n}\$ functions at \$n \geq 1\$ have no poles on the real axis while \$F\_0(p)\$ has two poles, at \$p = \pm k\_{10}\$. The \$F\_0(p)\$ function affects only the \$\zeta\_{zz}\$ component, which may be represented as

$$\begin{aligned} \zeta_{zz}(\vec{p}, \vec{p}') &= \frac{\zeta_0 k}{k_{10} ab} \sin^2(\alpha_1 x_0) \cos[k_{10}(z - z')] - \frac{i\zeta_0}{2\pi kb} \\ &\cdot \left\{ k^2 \int_{-\infty}^{\infty} dp F_0(p) \exp[ip(z - z')] \right. \\ &+ 2 \int_{-\infty}^{\infty} dp \sum_{n=1}^{\infty} F_{2n}(p) R_{2n}^{zz}(p; y, y') \\ &\cdot \left. \exp[ip(z - z')] \right\} \end{aligned} \quad (7)$$

where the symbol \$\int\$ stands for the principal value.

### III. SOLUTION

To calculate scattering matrix elements we apply the Galerkin method to the integral equation (2), putting

$$\begin{aligned} h_y(y, z) &= (U/\zeta_0) \sum_{\mu, \nu} A_{\mu\nu} \varphi_{\mu\nu}(u, v) \\ h_z(y, z) &= (U/\zeta_0) \sum_{\gamma, \eta} B_{\gamma\eta} \psi_{\gamma\eta}(u, v) \end{aligned} \quad (8)$$

where \$A\_{\mu\nu}\$ and \$B\_{\gamma\eta}\$ are unknown coefficients while \$u = (2y - b)/d\$ and \$v = 2z/l\$ are dimensionless variables. From here on summing with respect to \$\mu\$, \$\nu\$, \$\gamma\$, and \$\eta\$, one must take \$\mu = 1, 2, \dots, M\_y\$, \$\nu = 1, 2, \dots, N\_y\$, \$\gamma = 1, 2, \dots, M\_z\$, and \$\eta = 1, 2, \dots, N\_z\$. Choosing the basis functions, we should take into account the previously [8] obtained \$\bar{h}\$ behavior near the edge.

$$h_y \sim (1 - u^2)^\tau \quad h_z \sim (1 - u^2)^{1/2}, \quad u \rightarrow \pm 1, \quad (9a)$$

$$h_y \sim (1 - v^2)^{\tau/2} \quad h_z \sim (1 - v^2)^\tau, \quad v \rightarrow \pm 1, \quad (9b)$$

(\$\tau = -1/2\$ when \$W = 0\$, and \$\tau = 0\$ when \$W \neq 0\$) as well as the parity \$h\_y(-u, v) = -h\_y(u, v)\$ and \$h\_z(-u, v) = h\_z(u, v)\$, which follow from the fact that the incident mode field is not dependent on \$y\$ and the configuration in Fig. 1 is symmetrical with respect to the \$y = b/2\$ plane (see Section II). To calculate the scattering matrix it is sufficient to set only Fourier transforms of the basis functions rather than functions themselves. In accordance with [9], we assume

$$\begin{aligned} Y_{\mu\nu}(p_u, p_v) &\equiv \int_{-1}^1 du \int_{-1}^1 dv \varphi_{\mu\nu}(u, v) \sin(p_u u) \exp(-ip_v v) \\ &= J_{2\mu+\tau-1/2}(p_u) J_\nu(p_v) / p_u^{\tau+1/2} p_v \end{aligned}$$

$$\begin{aligned} Z_{\gamma\eta}(p_u, p_v) &\equiv \int_{-1}^1 du \int_{-1}^1 dv \psi_{\gamma\eta}(u, v) \cos(p_u u) \exp(-ip_v v) \\ &= J_{2\gamma-1}(p_u) J_{\eta+\tau-1/2}(p_v) / p_u p_v^{\tau+1/2} \end{aligned} \quad (10)$$

which corresponds to the use Gegenbauer's polynomials of the proper weight as the basis functions. As the result we get a system of linear algebraic equations (SLAE).

The scattering matrix elements sought are expressed in terms of the SLAE solution as follows.

$$\begin{aligned} S_{11} = S_{22} &= \frac{kld}{4k_{10} ab} \exp(ik_{10}l) \sum_{\gamma, \eta} (-1)^\eta B_{\gamma\eta} d_{\gamma\eta} \\ S_{12} = S_{21} &= \exp(ik_{10}l) \left[ 1 - \frac{kld}{4k_{10} ab} \sum_{\gamma, \eta} B_{\gamma\eta} d_{\gamma\eta} \right] \end{aligned} \quad (11)$$

where

$$d_{\gamma\eta} = \sin(\alpha_1 x_0) \delta_{1\gamma} J_{\eta+\tau-1/2}(k_{10}l/2) / (2(k_{10}l/2)^{\tau+1/2}). \quad (12)$$

For a waveguide wherein the RF is a lossy line,

$$|S_{11}|^2 + |S_{21}|^2 < 1. \quad (13)$$

The numerical results obtained are not considered to be true if the energy conservation law is not observed. For this reason, we calculate the ratio \$\chi\$ of the power absorbed by the RF to the incident mode power. Thus it is possible to control the equality

$$|S_{11}|^2 + |S_{21}|^2 + \chi = 1. \quad (14)$$

### IV. SHORT-FILM CASE

In all the regions of a short (\$l \ll d\$) film (except the small ones near its ends \$y = (b \pm d)/2\$), the relation \$|h\_z| \gg |h\_y|\$ is fulfilled and the \$h\_y\$ component may be neglected. As a

result, the vector integral equation (2) becomes scalar. We obtain an integral equation

$$Wh_z(\vec{\rho}) + \int d\sigma' \tilde{\zeta}(\vec{\rho}, \vec{\rho}') h_z(\vec{\rho}') = U \sin(\alpha_1 x_0) \quad (15)$$

where  $\tilde{\zeta}$  is an asymptotic expression of the kernel component  $\zeta_{zz}$  if in  $\zeta_{zz}$  and in  $\vec{E}_0(x, z)$  we leave terms not vanishing at  $l \rightarrow 0$ . To find  $\tilde{\zeta}$ , we express  $\zeta_{zz}$  as a double series.

$$\begin{aligned} \zeta_{zz}(\vec{\rho}, \vec{\rho}') = & \frac{\zeta_0}{kab} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \epsilon_n \frac{k^2 - \beta_{2n}^2}{k_{m,2n}} \sin^2(\alpha_m x_0) \\ & \cdot \cos(\beta_{2n} y) \cos(\beta_{2n} y') \exp[ik_{m,2n}|z - z'|]. \end{aligned} \quad (16)$$

Extracting the logarithmic singularity from the summation and neglecting the terms vanishing at  $l \rightarrow 0$ , we obtain

$$\begin{aligned} \zeta_{zz}(\vec{\rho}, \vec{\rho}') \approx & \tilde{\zeta}(\vec{\rho}, \vec{\rho}') = \frac{k\zeta_0}{k_{10}ab} \sin^2(\alpha_1 x_0) \\ & + i\zeta_0 \left\{ \frac{k}{\pi b} \sin^2(\alpha_1 x_0) + \frac{1}{2\pi k} \ln \left[ \frac{\pi|z - z'|}{2a \sin(\alpha_1 x_0)} \right] \right. \\ & \cdot \left( k^2 + \frac{\partial^2}{\partial y^2} \right) \delta(y - y') \\ & - \frac{1}{kab} \sum_{m=1}^{\infty} \epsilon_n (k^2 - \beta_{2n}^2) \sin^2(\alpha_m x_0) \cos(\beta_{2n} y) \\ & \cdot \cos(\beta_{2n} y') \left( \frac{1}{q_{m,2n}} - \frac{1}{\alpha_m} \right) \left. \right\} \end{aligned} \quad (17)$$

where  $q_{mn} = -ik_{mn}$ . The prime means that the term corresponding to  $m = 1, n = 0$  has been eliminated from the summation.

Further consideration will be based on the variational Schwinger method [10]. Using the integral equation (15), we construct a variational functional after Schwinger for the reflection coefficient  $S_{11}$ . This functional is stationary at the exact solution of (15). Using the asymptotic expression (17), we conclude as in [6] that a waveguide with a short film may be described by means of the equivalent circuit shown in Fig. 2. The scattering matrix elements become

$$S_{11} = -\exp(ik_{10}l) \frac{1}{2Z + 1} \quad S_{21} = \exp(ik_{10}l) \frac{2Z}{2Z + 1}. \quad (18)$$

The shunt impedance  $Z$  is represented as an amplitude-independent functional.

$$Z\{h_z\} = \frac{W \int d\sigma h_z^2(\vec{\rho}) + i \iint d\sigma d\sigma' h_z(\vec{\rho}) \text{Im} \tilde{\zeta}(\vec{\rho}, \vec{\rho}') h_z(\vec{\rho}')} {\frac{2\zeta_0 k}{k_{10}ab} \sin^2(\alpha_1 x_0) \left[ \int d\sigma h_z(\vec{\rho}) \right]^2}. \quad (19)$$

The calculation accuracy greatly depends upon the probe distribution of  $h_z$  being close to the true value. First of all, taking into account the edge conditions (9b), we express the  $h_z$  function in the form

$$h_z(\vec{\rho}) = (U/\zeta_0) [\pi^2(1 - v^2)/4]^\tau f(u). \quad (20)$$

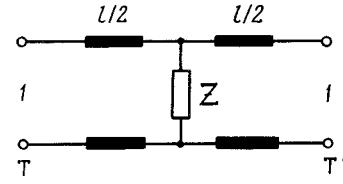


Fig. 2. Equivalent circuit for the short RF.

To choose the function  $f(u)$ , we substitute (20) into (15) and with respect to  $v$  we average both parts of the equation obtained. Leaving only logarithmic terms in the kernel of the averaged equation, we obtain a differential equation,

$$\begin{aligned} \left( -i \frac{4\delta}{k^2 d^2} \frac{\partial^2}{\partial u^2} + w - i\delta \right) f(u) &= \sin(\alpha_1 x_0) \\ \delta = \frac{kl}{2\pi} L & \quad L = \ln \left[ \frac{2 \sin(\alpha_1 x_0)}{\alpha_1 l} \right] + \begin{cases} 2\ln 2, & w = 0 \\ 3/2, & w \neq 0 \end{cases} \\ w = W/\zeta_0. & \end{aligned} \quad (21)$$

The solution of this equation being even with respect to  $u$  and vanishing at  $u = \pm 1$  may be written as

$$f(u) = \frac{\sin(\alpha_1 x_0)}{w - i\delta} \left[ 1 - \frac{\cos(\theta u)}{\cos \theta} \right], \quad \theta = \frac{kd}{2} \sqrt{1 + iw/\delta}. \quad (22)$$

In the limiting case  $w \rightarrow \infty$  (22) becomes  $f(u) \approx \sin(\alpha_1 x_0)/w$ , which is proportional to the solution of the integral equation (15) with the neglected integral term. In the second limiting case,  $w = 0$ , (22) describes a well-known current distribution on the logarithmically thin vibrator. Substituting (20) and (22) into (19), we obtain

$$\begin{aligned} Z = & \frac{wk_{10}ab}{2kldI_0 \sin^2(\alpha_1 x_0)} + i \frac{k_{10}}{\alpha_2 \sin^2(\alpha_1 x_0)} \\ & \cdot \left\{ \sin^2(\alpha_1 x_0) - \frac{b}{2dI_0} L \right. \\ & \left. - \sum_{m=1}^{\infty} \epsilon_n' \left[ 1 - \left( \frac{\beta_{2n}}{k} \right)^2 \right] \sin^2(\alpha_m x_0) \left( \frac{\alpha_1}{q_{m,2n}} - \frac{1}{m} \right) \left( \frac{I_n}{I_0} \right)^2 \right\} \end{aligned} \quad (23)$$

where

$$I_n = \frac{\theta^2 [\sin(\beta_n d)/\beta_n d] - \theta \tan \theta \cos(\beta_n d)}{\theta^2 - (\beta_n d)^2}.$$

When  $\sqrt{\delta/w} \ll 1$ , the shunt impedance may be described in a more simple way.

$$Z = \frac{wk_{10}ab}{2kld \sin^2(\alpha_1 x_0)} \left[ 1 + \frac{1+i}{kd} \sqrt{\frac{2\delta}{w}} \right]. \quad (24)$$

One can show that formulas (23) and (24) are also true for an RF placed in the waveguide cross section.

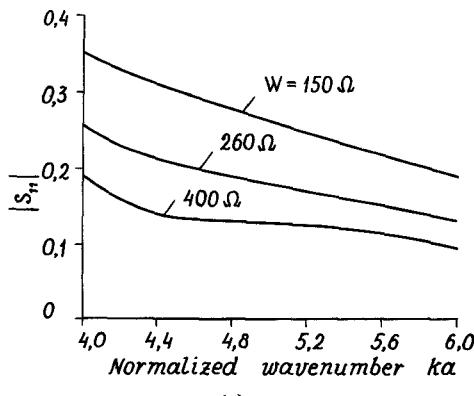
## V. NUMERICAL RESULTS

The computed results presented in Table I illustrate the stable character of convergence of the method for an increase of  $M_y = M_z \equiv M$  and  $N_y = N_z \equiv N$ . We stress the

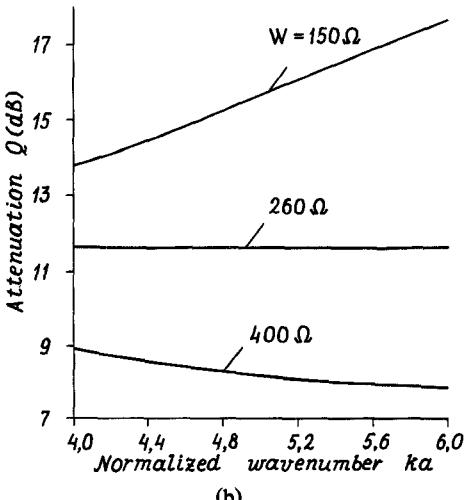
TABLE I  
CONVERGENCE OF THE GALERKIN METHOD IN CALCULATING THE SCATTERING MATRIX  
FOR THE DISCONTINUITY IN FIG. 1

$N$	$M = 1$	$M = 2$	$M = 3$	$M = 4$
$S_{11}$				
1	$-0.0251 - i \cdot 0.3539$	$-0.0270 - i \cdot 0.3521$	$-0.0270 - i \cdot 0.3520$	$-0.0270 - i \cdot 0.3520$
2	$-0.0755 - i \cdot 0.1951$	$-0.0724 - i \cdot 0.1932$	$-0.0723 - i \cdot 0.1934$	$-0.0723 - i \cdot 0.1934$
3	$-0.0662 - i \cdot 0.1959$	$-0.0638 - i \cdot 0.1944$	$-0.0639 - i \cdot 0.1945$	$-0.0639 - i \cdot 0.1945$
4	$-0.0670 - i \cdot 0.1978$	$-0.0644 - i \cdot 0.1965$	$-0.0644 - i \cdot 0.1966$	$-0.0644 - i \cdot 0.1966$
5	$-0.0674 - i \cdot 0.1977$	$-0.0649 - i \cdot 0.1962$	$-0.0650 - i \cdot 0.1963$	$-0.0650 - i \cdot 0.1963$
6	$-0.0675 - i \cdot 0.1977$	$-0.0650 - i \cdot 0.1963$	$-0.0651 - i \cdot 0.1964$	$-0.0651 - i \cdot 0.1964$
7	$-0.0676 - i \cdot 0.1977$	$-0.0650 - i \cdot 0.1963$	$-0.0651 - i \cdot 0.1964$	$-0.0651 - i \cdot 0.1964$
8	$-0.0676 - i \cdot 0.1977$	$-0.0651 - i \cdot 0.1963$	$-0.0651 - i \cdot 0.1964$	$-0.0651 - i \cdot 0.1964$
$S_{21}$				
1	$-0.3905 + i \cdot 0.5770$	$-0.3925 + i \cdot 0.5788$	$-0.3925 + i \cdot 0.5788$	$-0.3925 + i \cdot 0.5788$
2	$-0.3324 + i \cdot 0.4198$	$-0.3397 + i \cdot 0.4224$	$-0.3397 + i \cdot 0.4226$	$-0.3397 + i \cdot 0.4226$
3	$-0.3224 + i \cdot 0.4250$	$-0.3314 + i \cdot 0.4271$	$-0.3315 + i \cdot 0.4275$	$-0.3315 + i \cdot 0.4275$
4	$-0.3231 + i \cdot 0.4241$	$-0.3319 + i \cdot 0.4259$	$-0.3321 + i \cdot 0.4262$	$-0.3321 + i \cdot 0.4263$
5	$-0.3225 + i \cdot 0.4240$	$-0.3312 + i \cdot 0.4258$	$-0.3313 + i \cdot 0.4261$	$-0.3313 + i \cdot 0.4262$
6	$-0.3226 + i \cdot 0.4239$	$-0.3313 + i \cdot 0.4257$	$-0.3315 + i \cdot 0.4261$	$-0.3315 + i \cdot 0.4262$
7	$-0.3225 + i \cdot 0.4239$	$-0.3312 + i \cdot 0.4257$	$-0.3314 + i \cdot 0.4261$	$-0.3314 + i \cdot 0.4261$
8	$-0.3225 + i \cdot 0.4239$	$-0.3313 + i \cdot 0.4257$	$-0.3314 + i \cdot 0.4260$	$-0.3314 + i \cdot 0.4251$

$b/a = 0.5$ ,  $x_0/a = 0.5$ ,  $d/a = 0.4$ ,  $l/a = 0.5$ ,  $ka = 5$ , and  $W = 200 \Omega$ .

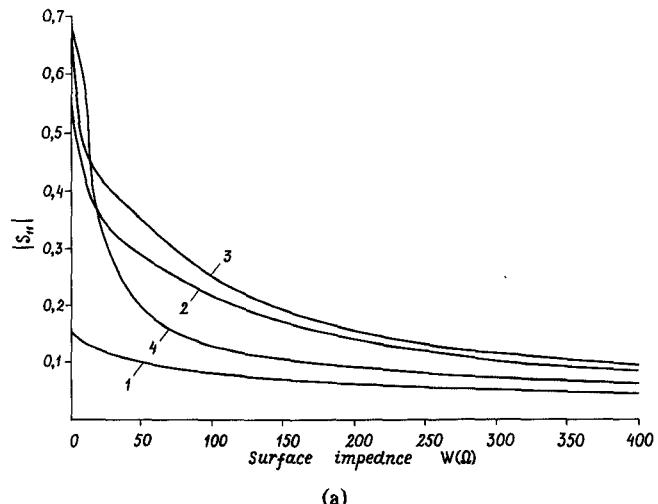


(a)

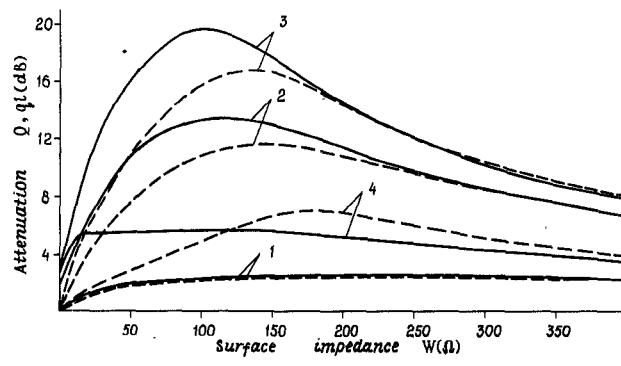


(b)

Fig. 3.  $|S_{11}|$  and  $Q$  versus frequency at  $b/a = 0.5$ ,  $d/a = 0.45$ ,  $l/a = 1$ , and  $x_0/a = 0.5$ .



(a)



(b)

Fig. 4.  $|S_{11}|$ ,  $Q$  (solid curves) and  $q$  (dashed curves) versus surface impedance  $W$  at  $b/a = 0.5$ ,  $l/a = 1$ ,  $ka = 6$ . Curves 1-3 correspond to  $x_0/a = 0.5$  and  $d/a = 0.2$ ,  $0.4$ , and  $0.45$ ; curve 4 corresponds to  $x_0/a = 0.25$  and  $d/a = 0.45$ .

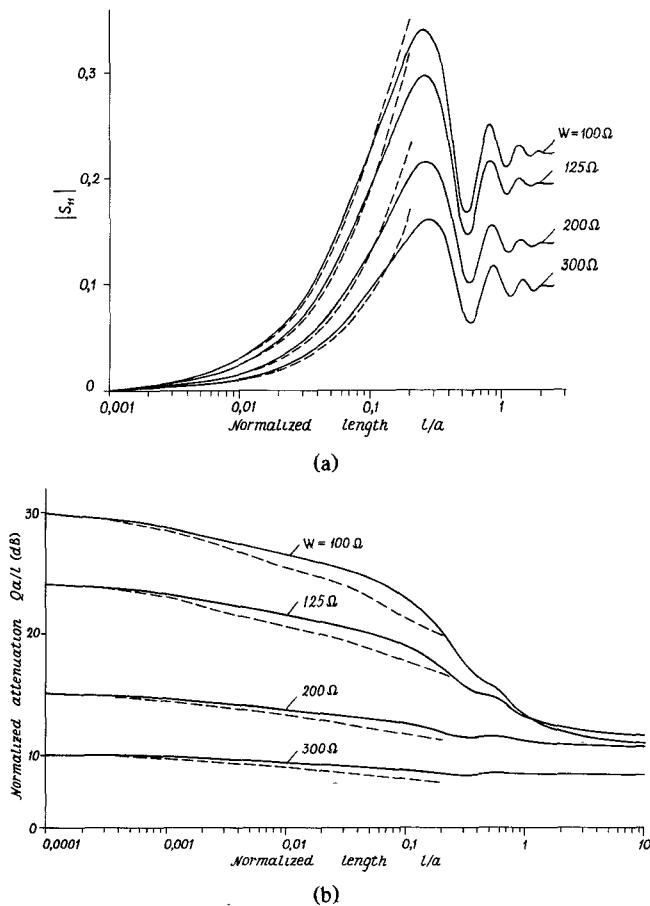


Fig. 5.  $|S_{11}|$ ,  $Q$  versus the film length  $l$  at  $b/a = 0.5$ ,  $d/a = 0.4$ ,  $x_0/a = 0.5$ ,  $ka = 6$ . Solid curves are calculated by (12); dashed curves are calculated by approximate formulas (18) and (23).

energy balance (14) to be true for any  $M_y$ ,  $N_y$ ,  $M_z$ , and  $N_z$  values with accuracy not less than eight digits.

Studies of the frequency dependencies of scattering characteristics are important for practical applications. The frequency dependencies of  $|S_{11}|$  and  $Q = -20 \log |S_{21}|$ , calculated for fixed dimensions of the RF and various  $W$  values, are shown in Fig. 3. It is evident that  $|S_{11}|$  decreases with  $k$  at the parameters given while  $Q$  may either decrease or increase. It turns out that the value of  $W = \bar{W}$  exists when  $Q$  does not actually depend upon the frequency ( $\bar{W} = 260 \Omega$  for the RF dimensions given). Until recently the fact of attenuation being independent of the frequency in the broad band was unknown. This is of a great practical value in attenuator design.

The  $|S_{11}|$  and  $Q$  dependencies upon  $W$  for various  $d/a$  ratios are given in Fig. 4. This figure shows that  $|S_{11}|$  decreases monotonically with  $W$  while the  $Q(W)$  dependence at  $d/a \geq 0.4$  has a clearly expressed maximum. To compare, we present  $ql$  versus  $W$ , where  $q$  is the dominant mode linear attenuation in a rectangular waveguide with the RF in the  $E$  plane [11].

The  $|S_{11}|$  and  $Qa/l$  dependencies upon the RF length  $l$  are shown in Fig. 5. One can see here that the dependence of  $|S_{11}|$  on  $l$  is oscillatory. This can be explained by the interaction between the ends of the film by means of the dominant mode. When the film is sufficiently long,  $|S_{11}|$  no longer depend on  $l$  and represents the reflection coefficient

from the semi-infinite film. Comparison of the exact and approximate data (Fig. 5(a) and (b)) shows that the accuracy of the approximate formulas is sufficient in practice for  $l/a < 0.01$ .

## VI. CONCLUSION

This is believed to be the first calculation of the characteristics of  $TE_{10}$  mode scattering by the RF of arbitrary width and length placed as shown in Fig. 1. The developed rigorous theory makes it possible to calculate the scattering matrix elements with high accuracy given arbitrary parameters. The previously unknown property of the frequency independence of the insertion attenuation over a broad band has been discovered for particular values of film width and surface impedance. This property is of a great practical importance.

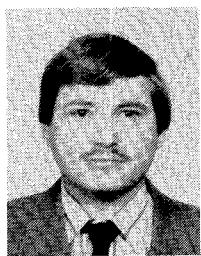
We may conclude on the basis of the experiments described in [1] that the scattering characteristics of discontinuities of the type investigated in this paper tend to display a resonance dependence on the parameters. Such dependences are rather complicated and their study requires an enormous number of calculations. Therefore, the results of our study of the RF resonance properties will be presented in a subsequent paper.

## ACKNOWLEDGMENT

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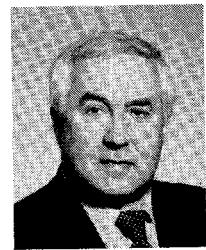
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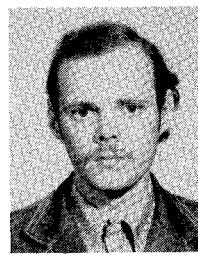
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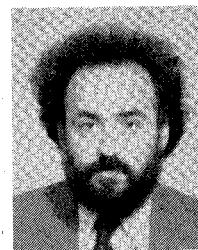
At his initiative, the manuscript of Shwinger's notes dealing with discontinuities in waveguides was brought from the United States in 1969 and published in Russian under his editorship in March 1970. From 1961 to 1978 he was Head of the Laboratory of Theoretical Investigation at the Vilnius Scientific Research Institute of Radio Measuring Devices. In 1978 he joined Grodno State University, where he was named Professor. Since 1989 he has been Head of the Department of Theoretical Radiophysics at the Zondas Company, Vilnius. Dr. Fridberg has published 90 works on applied electrodynamics in leading Soviet and American scientific journals and is the holder of 12 patents.



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